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A finite element method for computing natural frequencies of a submerged structure is implemented using a structural analysis program to solve a sample problem. In this method, the structure and the surrounding fluid are represented by finite elements. Finite element solutions of the sample problem compare well with an analytic solution for both consistent and lumped formulations of the fluid-structure interaction effects.

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Finite Element Solution of Fluid-Structure Interaction Problems

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ABSTRACT

A finite element method for computing natural frequencies of a submerged structure is implemented using a structural analysis program to solve a sample problem. In this method, the structure and the surrounding fluid are represented by finite elements. Finite element solutions of the sample problem compare well with an analytic solution for both consistent and lumped formulations of the fluid-structure interaction effects.

INTRODUCTION

The method of Zienkiewicz and Newton[1] for solving fluid-structure interaction problems, one of several developed to address such problems, uses finite elements to represent both the structure and the fluid; in the fluid the unknown is the pressure field. We used the two-dimensional problem of finding natural frequencies of a ring surrounded by fluid to explore the characteristics of this method and the practicality of using a finite element structural analysis program to apply it. Although this paper deals with only two-dimensional problems, the finite element method is applicable to three-dimensional problems involving generally shaped structures, and eventually we intend to explore such applications and transient problems as well.

In this investigation we considered the amount of fluid required to obtain adequate modeling, the effect of element size on accuracy, and the

[1] Zienkiewicz, O.C. and Newton, R.E., Coupled Vibrations of a Structure Submerged in a Compressible Fluid, Proc. Int. Symp. on Finite Element Techniques, Stuttgart, 1969.

ways in which the method handles various vibration modes. We also compared a lumped representation of the fluid-structure interaction with a consistent representation.

An extension of the method of Zienkiewicz and Newton was used by Hunt et al[2] to determine the acoustic radiation from a piezoelectric cylinder. Everstine et al[3] reviewed this and other methods for fluid-structure interaction problems and point out advantages and disadvantages among them.

A FINITE ELEMENT REPRESENTATION OF A VIBRATING FLUID-STRUCTURE SYSTEM

The method of Zienkiewicz and Newton represents both the structure and the fluid with finite elements. The finite elements determine displacements in the structure and a pressure field in the fluid. These representations produce one system of linear algebraic equations for the structure and a second for the fluid. The two systems are coupled by terms resulting from dynamic interactions at the fluid-structure boundary. Solving an algebraic eigenvalue problem yields the natural frequencies of the fluid-structure system.

The system of linear algebraic equations that represents the structure is written in the form of a matrix equation

$$M\ddot{\delta} + K\delta = f \tag{1}$$

where

- δ and $\ddot{\delta}$ are vectors of displacements and accelerations at grid points in the structure,
- M and K are mass and stiffness matrices for the structure, and
- f is a vector of generalized external forces acting on the structure.

Each component f_i of the vector f of generalized forces corresponds to a displacement δ_i at a grid point a in the structure. If the point a is on the surface S_s of the structure that is acted on by the fluid pressure p,

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[2] Hunt, J.T., Knittel, M.R., and Barach, D., Finite Element Approach to Acoustic Radiation from Elastic Structures, J. Acoust. Soc. Am., pp. 269-280, Vol. 55, 1974.

[3] Everstine, G.C., Schroeder, E.A., and Marcus, M.S., The Dynamic Analysis of Submerged Structures, NASTRAN: User's Experiences, Colloquium held at Langley Research Center, Hampton, Virginia, September 1975.

the generalized force component is given by

$$f_i = \int_{S_s} N_i n_i p \, dS \tag{2}$$

where N_i is the shape function corresponding to δ_i, and n_i is the component in the direction of the displacement δ_i of the unit vector $\hat{n}$ normal to S_s and directed out of the fluid region. The fluid pressure is expressed as a linear combination of shape functions F_j defined in the region containing fluid

$$p = \sum_j \psi_j F_j \tag{3}$$

in which each ψ_j represents the pressure at a grid point in the fluid. With inner product notation used for conciseness, the component of generalized force maybe written in the form

$$f_i = \sum_j \psi_j < N_i', F_j> \tag{4}$$

where

$$N_i' = n_i N_i \tag{5}$$

Equation (4) is written in matrix form using the "boundary matrix " B,

$$f = B\psi \tag{6}$$

where ψ is the vector of fluid pressures at grid points in the fluid, and the consistent formulation of the boundary matrix is

$$B = [<N_i', F_j>] \tag{7}$$

An alternative to the consistent formulation of the boundary matrix is the lumped formulation. To evaluate the integral in Equation (2) for the lumped formulation, the fluid pressure is assumed to be constant over the element faces e_k lying on the fluid-structure boundary that contain the grid point a. If the constant value is taken to be the pressure at the point a, Equation (2) becomes

$$f_i = \psi_i n_i \sum \int_{e_k} N_i \, dS$$

The sum is taken over all element faces e_k connected to the grid point a. When the configuration of the element faces and the shape functions are given, this sum can be computed. For the lumped formulation, the boundary matrix is diagonal:

$$B = \mathrm{diag}[n_i \sum \int_{e_k} N_i \, dS] \tag{8}$$

Using the boundary matrix B in either the consistent or the lumped formulation, we substitute Equation (6) into Equation (1). Then the finite element matrix equation for the structure including the effects of fluid pressure on the fluid-structure boundary is

$$M\ddot{\delta} + K\delta - B\psi = 0 \tag{9}$$

A similar matrix equation is needed to represent the pressure field in the fluid. To obtain a finite element matrix equation for the fluid, the fluid is assumed to be an acoustic medium; that is, it has a pressure field p defined by the wave equation

$$\frac{1}{c^2}\ddot{p} - \nabla^2 p = 0 \tag{10}$$

where c is the speed of sound in the fluid. The pressure field is expanded in terms of shape functions (Equation (2)), and the Galerkin method is applied to the wave equation (Reference 1) to get the following equation:

$$Q\ddot{\psi} + H\psi - \{ \int_S F_i \frac{\partial p}{\partial \hat{n}} dS \} = 0 \tag{11}$$

where $\qquad Q = [\frac{1}{c^2} \langle F_i, F_j \rangle] \qquad$ and $\qquad H = [\langle \nabla F_i, \nabla F_j \rangle]$.

The boundary term appears in the i^{th} equation of the system, Equation (11), only if ψ_i is the pressure at a grid point on the boundary of the fluid region. The integral in the boundary term is the sum of integrals contributed by various sections of the fluid region boundary. Equation (11) will be the required matrix equation for the fluid after the form of the boundary term is converted to a matrix multiplied by the vector $\ddot{\delta}$.

The contribution of the fluid-structure boundary to this boundary term is due to the effect of the structure's motion on the fluid pressure. On the fluid-structure boundary, the boundary condition is

$$\frac{\partial p}{\partial \hat{n}} = - \rho_f \ddot{\delta}_{\hat{n}}$$

where $\hat{n}$ is the unit vector normal to the boundary, $\ddot{\delta}_{\hat{n}}$ is the normal component of the boundary's acceleration, and ρ_f is the fluid's density. With the shape functions N_i' (Equation (5)) which incorporate components of $\hat{n}$, the acceleration of the structure's boundary can be expressed by

$$\ddot{\delta}_{\hat{n}} = \sum_i N_i' \ddot{\delta}_i$$

Therefore the contribution of the fluid-structure boundary to the boundary term is

$$\rho_f \sum_i \ddot{\delta}_i \langle N_i', F_j \rangle = \rho_f B^T \ddot{\delta} \qquad (12)$$

The remaining contributions to the boundary term in Equation (11), due to boundaries of the fluid region other than the fluid-structure boundary, are zero as is next shown. The finite element method used here represents with finite elements only a finite portion of a fluid that extends to infinity and all significant changes in the fluid region are assumed to occur within this finite region. This assumption is reflected by setting either the pressure or the normal derivative of the pressure equal to zero on the outer boundary of the fluid region. (The method as presented here makes no provisions for radiation from the outer boundary.) On the sections of the fluid boundary formed by a plane of symmetry, the normal derivative $\partial p/\partial \hat{n}$ equals zero; on the sections bounded by a plane of anti-symmetry the pressure p equals zero. Therefore, on these boundaries either $\partial p/\partial \hat{n} = 0$ and the boundary term is zero, or ψ_i is constrained to equal zero. Constraining ψ_i to be zero on the boundary has the effect of removing the i^{th} equation from Equation (11). Thus on these boundaries the contribution of the boundary term is zero.

From the preceding remark and from Equation (12) the fluid finite element matrix equation, which includes effects of motion of the structure on the fluid pressure, is

$$Q\ddot{\psi} + H\psi + \rho_f B^T \ddot{\delta} = 0 \qquad (13)$$

As stated before, the boundary matrix may be computed by either the consistent or lumped formulation.

Finally, combining Equations (9) and (13) gives the matrix finite element equation for the fluid-structure system

$$\begin{bmatrix} M & 0 \\ \rho_f B^T & Q \end{bmatrix} \begin{Bmatrix} \ddot{\delta} \\ \ddot{\psi} \end{Bmatrix} + \begin{bmatrix} K & -B \\ 0 & H \end{bmatrix} \begin{Bmatrix} \delta \\ \psi \end{Bmatrix} = 0$$

If a sinusoidal time dependence with frequency ω is assumed, so that $\ddot{\delta} = -\omega^2\delta$ and $\ddot{\psi} = -\omega^2\psi$, the preceding matrix equation becomes the matrix eigenvalue problem:

$$\det \left\{ \omega^2 \begin{bmatrix} M & 0 \\ \rho_f B^T & Q \end{bmatrix} - \begin{bmatrix} K & -B \\ 0 & H \end{bmatrix} \right\} = 0 \qquad (14)$$

The roots ω of this equation are the natural frequencies of the fluid-structure system.

To set up and solve this eigenvalue problem, we used the digital computer program NASTRAN. NASTRAN can be manipulated to compute the submatrices, assemble the combined problem, and solve for the natural frequencies.

MODELING FLUID-STRUCTURE SYSTEMS USING STRUCTURAL FINITE ELEMENTS

This finite element method for representing vibrations of fluid-structure systems uses standard structural finite elements to represent the submerged structure and adapted structural elements to represent the surrounding fluid. Although the simple problem used here represents a two-dimensional system, a similar method can be applied to three-dimensional systems.

In two dimensions, the fluid is represented by elastic membrane elements in which one component of displacement is identified with the fluid pressure, the other component is constrained to equal zero, and the material parameters are chosen so that the resulting pressure field satisfies the wave equation. To compute the entries in a consistent boundary matrix, it is convenient to use modified extensional rods connected between grid points on the fluid-structure boundary. The entries in a lumped boundary matrix are precomputed and entered directly.

The finite element representation of the structure is formed by the usual methods and is not elaborated upon here. This representation of the structure produces the matrices M and K for Equation (14).

The finite element representation of the fluid region is formed by analogy using elastic membrane elements. For an acoustic medium, the two-dimensional pressure field p (force/unit length) satisfies the wave equation

$$\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} - \frac{\partial^2 p}{\partial y^2} = 0 \qquad (15)$$

The displacements of the elastic membrane elements are u and v in the x and y directions. The differential equations for planar motions of the membrane are determined by the two-dimensional elastic modulus matrix for anisotropic material

$$G_m = \begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix}$$

The anisotropic matrix is used for its flexibility in choosing material parameters. In general, if the orientation of a finite element is such that the element forms an angle θ with the anisotropic material axis, the elastic modulus matrix for the element is obtained by the transformation

$$G_e = U^T G_m U$$

where

$$T = \begin{bmatrix} \cos^2\theta & \sin^2\theta & \cos\theta\sin\theta \\ \sin^2\theta & \cos^2\theta & -\cos\theta\sin\theta \\ -2\cos\theta\sin\theta & 2\cos\theta\sin\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

However, the fluid is isotropic, and since fluid elements may have various orientations, it is convenient for the matrix G_e to be invariant under these transformations. It is straightforward to verify that G_e is invariant if

$$G_{11} = G_{22}, \quad 2G_{33} = G_{11} - G_{12}, \quad G_{13} = G_{23} = 0$$

Therefore, the elastic modulus matrix is of the form

$$G = \begin{bmatrix} G_1 & G_2 & 0 \\ G_2 & G_1 & 0 \\ 0 & 0 & \frac{G_1 - G_2}{2} \end{bmatrix}$$

The differential equations for the membrane become

$$\rho_m \frac{\partial^2 u}{\partial t^2} = G_1 \frac{\partial^2 u}{\partial x^2} + \frac{G_1 + G_2}{2} \frac{\partial^2 v}{\partial x \partial y} + \frac{G_1 - G_2}{2} \frac{\partial^2 u}{\partial y^2}$$

$$\rho_m \frac{\partial^2 v}{\partial t^2} = \frac{G_1 + G_2}{2} \frac{\partial^2 u}{\partial x \partial y} + \frac{G_1 - G_2}{2} \frac{\partial^2 v}{\partial x^2} + G_1 \frac{\partial^2 v}{\partial y^2}$$

where ρ_m is the area density of the membrane. The finite element method approximates these differential equations by a system of linear algebraic equations. Constraining v to be a constant removes from the algebraic system those equations that correspond to the second differential equation, therefore u needs to satisfy only the first differential equation, which then becomes the wave equation (Equation (1b)) if u is identified with the fluid pressure p and the membrane material parameters are set as follows:

$$\rho_m = 1/c^2, \quad G_1 = 1, \quad G_2 = -1$$

Therefore the mass and stiffness matrices produced by the membrane elements provide Q and H for Equation (14).

For the consistent formulation, the boundary matrix (see Equation (7))

$$B = [\langle N_i', F_j \rangle]$$

represents interactions at the fluid-structure boundary. We will consider the contribution to B from a fluid-structure boundary segment between grid points a and b. Let δ_{ax} and δ_{ay} be the displacements in the x and y directions and p_a and p_b be the pressures at grid points a and b. The submatrix of B that couples these displacements with these pressures is

$$\begin{bmatrix} n_x\langle N_{ax}, F_a\rangle & n_x\langle N_{ax}, F_b\rangle \\ n_y\langle N_{ay}, F_a\rangle & n_y\langle N_{ay}, F_b\rangle \end{bmatrix}$$

In these terms, the integration is restricted to the segment between a and b. A similar matrix couples δ_{bx} and δ_{by} with p_a and p_b. Since the entries in these matrices are similar to entries in the mass matrix of an extensional rod connecting a and b, a minor modification of an extensional rod element will produce these entries for the boundary matrix B in Equation (14).

To illustrate the computation of entries in the boundary matrix for the lumped formulation, we used a typical case in which two segments of a two-dimensional fluid-structure boundary join at the grid point b, as shown in Figure 1.

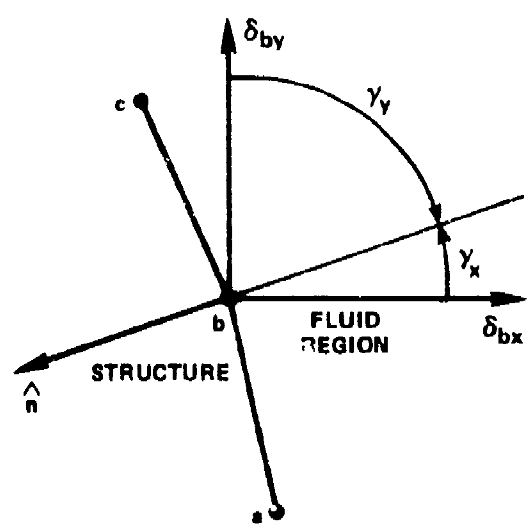

Figure 1 – A Typical Section of a Fluid Structure Boundary

For the lumped formulation, the fluid pressure on the surface segments connected to the grid point b is taken to equal the pressure p_b at the grid point. Also the unit vector $\hat{n}$ normal to the fluid-structure boundary at b is taken to be the unit vector whose direction is midway between the normal vectors on either side of b. Then the generalized force applied to b in the direction of δ_{bx} is approximated by (see Equation (2))

$$p_b n_x \int_a^c N_{bx} \, ds$$

where n_x is the component of $\hat{n}$ in the direction δ_{bx}, and N_{bx} is the shape function corresponding to δ_{bx}. Recall that in the two-dimensional case p has dimensions: force/unit length. If the average length of the segments from a to b and from b to c is L, this approximation becomes

$$p_b L \cos \gamma_x$$

where γ_x is the angle between $\hat{n}$ and the direction of δ_{bx}. Similarly, the generalized force applied to b in the direction of δ_{bx} is approximated by

$$p_b L \cos \gamma_y$$

Therefore, the contribution to the boundary matrix that couples δ_{bx} and δ_{by} with p_b is

$$[\ L \cos \gamma_x \quad L \cos \gamma_y\ ] \tag{16}$$

These numbers are precomputed for each fluid-structure boundary grid point and entered directly into the fluid-structure matrix equation.

MODELING FLUID-STRUCTURE SYSTEMS USING A STRUCTURAL ANALYSIS PROGRAM

A general purpose finite element structural analysis computer program can be used to model the structure with the usual structural elements and the surrounding fluid with adapted membrane finite elements. The input data which describe the material properties, coordinate directions, and degrees of freedom associated with the membrane element set are tailored to the fluid analogy.

For the consistent formulation, a third set of finite elements (boundary elements) is introduced at the fluid-structure interface. A schematic representation of a section of the fluid-structure boundary is shown in Figure 2.

In implementing the consistent formulation, an option to compute the fluid-structure coupling matrix B is added (through minor program modification) to the subroutine which normally computes the consistent mass matrix for the boundary finite elements.

Thus, from the configuration of structural, fluid, and boundary elements, the following block diagonal composite mass and stiffness matrices are formed:

$$\begin{bmatrix} M & & \\ & B & \\ & & Q \end{bmatrix} \qquad \begin{bmatrix} K & & \\ & D & \\ & & H \end{bmatrix} .$$

Then, options typically available in finite element computer programs with matrix interpretive capabilities are used to interrupt the normal flow of the standard eigenvalue analysis to partition, reduce, and rearrange the composite matrices to form the matrices required for Equation (14). (The stiffness matrix D associated with the boundary elements is discarded.) Then the normal flow of the eigenvalue analysis is resumed.

In implementing the lumped version, the coupling terms $L \cos \gamma_x$ and $L \cos \gamma_y$ (as derived for Equation (16)) are precomputed and inserted in the stiffness matrix for the fluid-structure system in the column corresponding to the pressure variable of the fluid grid point b_{f_i}, and in rows corresponding to the structural displacements in the x- and y-coordinate directions, respectively, of the structural grid point b_{s_i}. (The spatial grid point b is represented as b_{s_i} and b_{f_i} within the sets of structural and fluid grid points, respectively, which lie at the fluid-structure boundary.) Similarly, the terms $-\rho_f \cdot L \cos \gamma_x$ and $-\rho_f \cdot L \cos \gamma_y$ are precomputed and inserted in the mass matrix for the fluid-structure system in the row corresponding to the pressure variable of the fluid grid point b_{f_i}, and in columns corresponding to the structural displacements in the x- and y-coordinate directions, respectively, of the structural grid point b_{s_i}. This coupling condition may be implemented directly through the input data to finite element structural analysis programs such as NASTRAN.

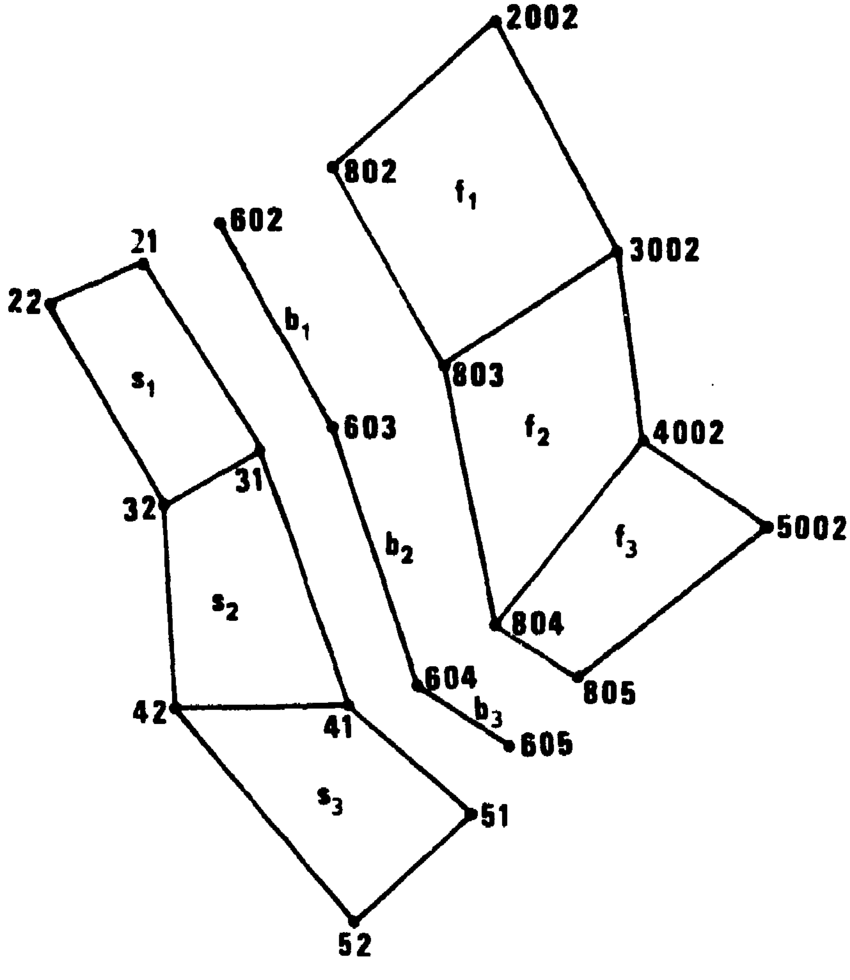

Figure 2 – Schematic Representation of a Section of the Fluid-Structure Boundary

s_i: structural elements, b_i: boundary elements, f_i: fluid elements

Grid points 21, 602, 802; 31, 603, 803; etc., have the same spatial coordinates. The boundary elements b_i are present for the consistent version only.

COMPARISON OF FINITE ELEMENT RESULTS WITH ANALYTIC SOLUTION FOR A SAMPLE PROBLEM

To study the effectiveness of the finite element pressure formulation, the two-dimensional vibrations of a ring surrounded by a compressible fluid were computed using the structural analysis computer program NASTRAN. The results of both consistent and lumped formulations were compared with an analytic solution.[4] The steel ring, 25.4 cm in radius and 0.635 cm thick (and of unit depth), vibrates in water.

The finite element mesh consists of $(N_\theta + 1)x(N_r + 1)$ spatial grid points which are determined by the intersection of $(N_r + 1)$ concentric circles with $(N_\theta + 1)$ lines radiating at N_θ equal intervals of the cylindrical coordinate θ. The ring, which is represented by the innermost circle, is modeled by N_θ bar elements, the fluid by $(N_\theta \times N_r)$ quadrilateral membrane elements. For the consistent formulation only, N_θ rod elements are added to the model at the fluid-structure boundary. To optimize numerical accuracy, the aspect ratio (ratio of longest to shortest side) of the fluid elements was kept as near unity as possible. Because of its symmetry, the problem may be modeled using only the first quadrant. A mesh with $N_\theta = 8$ and $N_r = 6$ is shown in Figure 3.

The finite element frequencies obtained for several even modes and the corresponding analytic values are compared in Table 1. Computer generated plots showing deformation patterns (mode shapes) of the structure and contour lines of constant pressure in the scalar fluid pressure field were obtained using NASTRAN plotting options. These plots resulted from solutions obtained using real eigenvalue analysis. Figure 4 shows plots obtained with the consistent formulation for the case $N_\theta = 8$, $N_r = 6$, and the boundary condition $p = 0$ applied at an outer radius of 82.8 cm. For each of the four plots corresponding to modes 0, 2, 4, and 6, the mode shapes appear in the lower left corners, underlayered by the undeformed ring. To the right are contour plots showing lines of constant pressure corresponding to eleven equally spaced values of pressure ranging from the minimum (label 1) to the maximum (label 11) value in the pressure field solution.

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[4] Schroeder, E.A. and Marcus, M.S., Natural Frequencies of a Submerged Ring, Computation and Mathematics Departmental Report CMD-27-74, David W. Taylor Naval Ship Research and Development Center (1974).

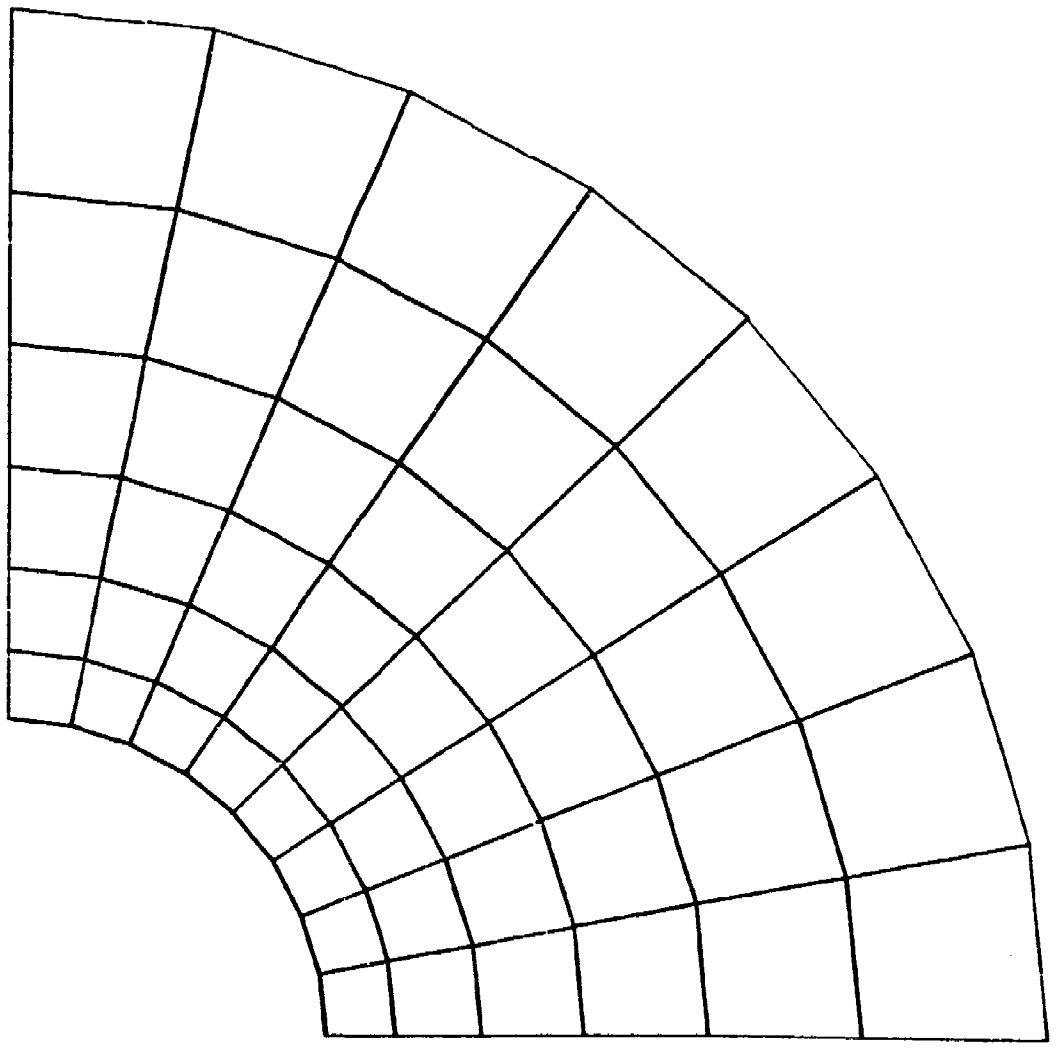

Figure 3 – Finite Element Mesh for Sample Problem

TABLE 1 - NATURAL FREQUENCIES OF THE RING
(Radians per second)

Analytic Results	Modes			
	0	2	4	6
no fluid, vacuum at 25.4 cm	[illegible]5	392.0	2126	5044
finite fluid, vacuum at 55.[illegible]	6680	231.2	1433	3725
finite fluid, vacuum at 68.[illegible]	5389	227.7	1432	3724
finite fluid, vacuum at 82.8 cm	4364	226.1	1432	3724
infinite fluid, no incoming waves	18191	224.7	1432	3724

FINITE ELEMENT RESULTS (Vacuum at Outer Radius)

N_θ	N_r	Outer Radius (cm)	FE Formulation	Modes			
				0	2	4	6
8	0	25.4	consistent	20277	393.9	2137	5074
8	4	55.9	consistent	6642	240.5	1563	4221
8	5	68.1	consistent	5353	237.0	1562	4221
8	6	82.8	consistent	4340	235.4	1562	4221
8	4	55.9	consistent	6642	240.5	1563	4221
16	8	55.9	consistent	6672	233.5	1468	3869
32	16	55.9	consistent	6680	231.8	1442	3762
8	6	82.8	consistent	4340	235.4	1562	4221
16	12	82.8	consistent	4357	228.4	1466	3869
32	24	82.8	consistent	4362	226.6	1441	3762
8	6	82.8	lumped	4333	231.6	1486	3884
16	12	82.8	lumped	4356	227.5	1447	3776
32	24	82.8	lumped	4362	226.4	1436	3738

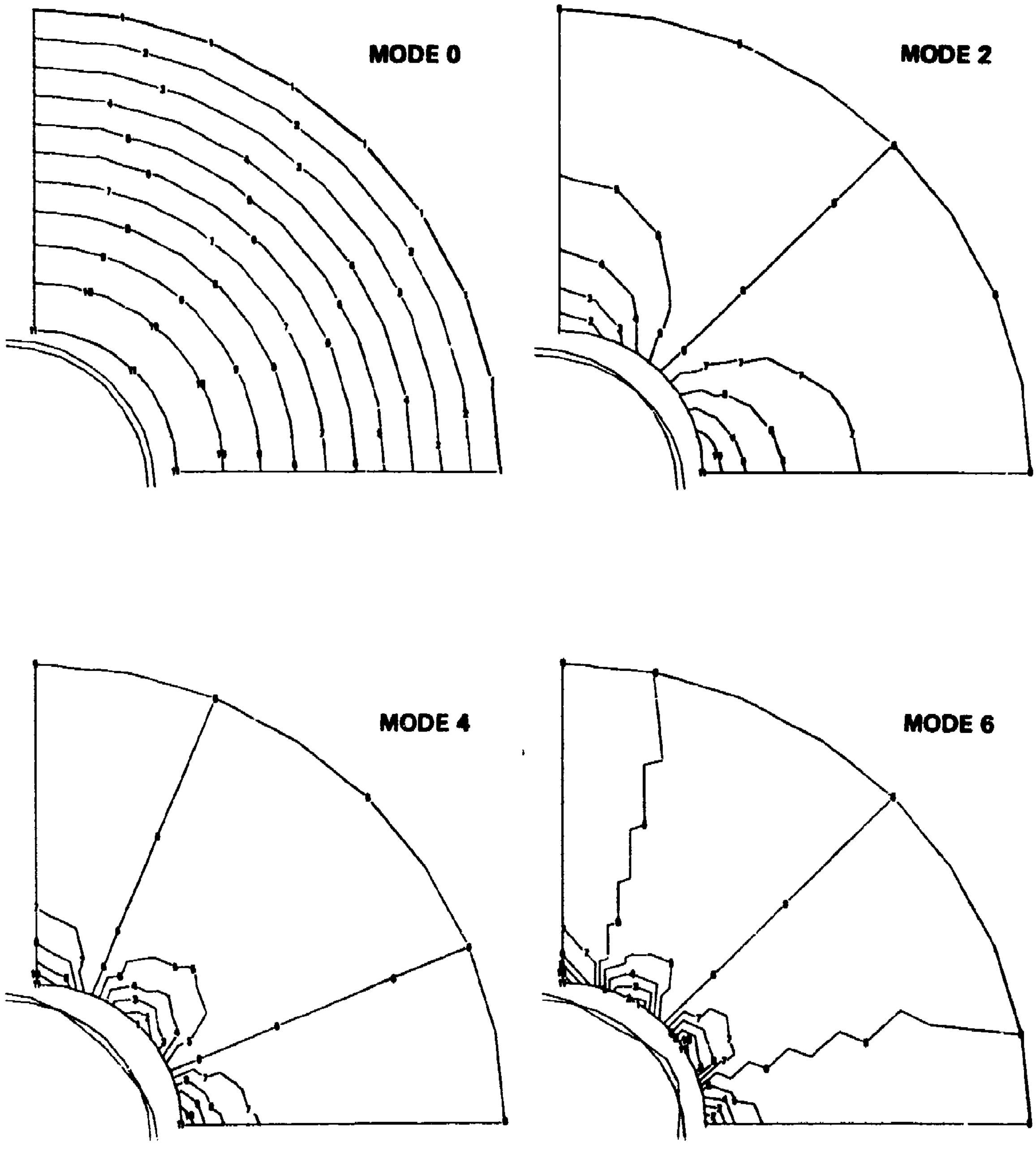

Figure 4 – Deformation and Pressure Contours

DISCUSSION OF RESULTS

Use of NASTRAN to implement the finite element pressure formulation by using structural elements to serve, by analogy, as fluid elements proceeded smoothly. Altering NASTRAN's standard procedures to assemble the mass and stiffness matrices for the combined fluid-structure problem and to insert the lumped boundary matrix presented no problem. The extensional rod element was modified to compute the consistent boundary matrix without difficulty.

Table 1 shows that, for modes 2, 4, and 6, the analytic solution for fluid regions with finite radii converges quickly to the solution for an infinite fluid region. But for mode 0 no such convergence is indicated. The same behavior is evident for the finite element solution for meshes with $N_\theta = 8$, for which the vacuum boundary condition (p=0) was imposed at the successively larger radii, 55.9 cm, 68.1 cm, and 82.8 cm. These trials show that extending the radius of the fluid region from 2.2 to 2.7 structural radii changes the frequencies of modes 4 and 6 by less than one percent. Extending the fluid radius from 2.7 to 3.3 structural radii changes the frequency of mode 2 by less than one percent.

Effects of element size on accuracy were studied by successively refining the finite element grid for fluid regions with radii 55.9 cm and 82.8 cm. The frequencies for these trials are given in Table 1 and the percent of deviation from the analytic solutions for the same radii are given in Table 2. In all cases the error is seen to decrease approximately as the square of the element size.

Accuracy of the frequencies relative to the frequencies obtained analytically and their convergence to the frequency of the infinite fluid region also depends on the mode of vibration. The higher mode required a more refined finite element grid than the lower modes to achieve equal accuracy. On the other hand, the frequencies of higher modes converge at smaller radii than do those of the lower modes. Although mode 0 does not converge at all as the radius increases, the finite element method gives an accurate estimate of the analytic frequency for equal radii.

For this problem, the lumped formulation of the fluid-structure boundary interaction produced results more accurate than the consistent formulation. Also,

TABLE 2 - PERCENTAGE ERRORS FOR FINITE FLUID NUMERICAL RESULTS

N_θ	N_r	Outer Radius (cm)	FE Formulation	Modes 0	2	4	6
8	4	55.9	consistent	0.6	4.0	9.1	13.3
16	8	55.9	consistent	0.1	1.0	2.4	3.9
32	16	55.9	consistent	0.0	0.3	0.6	1.0
8	6	82.8	consistent	0.6	4.1	9.1	13.3
16	12	82.8	consistent	0.2	1.0	2.4	3.9
32	24	82.8	consistent	0.0	0.2	0.6	1.0
8	6	82.8	lumped	0.7	2.4	3.7	4.3
16	12	82.8	lumped	0.1	0.6	1.0	1.4
32	24	82.8	lumped	0.0	0.1	0.3	0.4

the accuracy remained higher as the mode increased. These results suggest that a lumped formulation should be considered for a fluid-structure interaction problem unless the complexity of the problem precludes its use.

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Dr. Gordon Everstine suggested trying a lumped formulation for the fluid-structure boundary interactions and contributed many useful ideas toward forming the analog fluid elements. Mr. Myles Hurwitz provided expert consultation on implementing NASTRAN. The authors are grateful for their contributions.

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